Digital << ...

21/3/2016 الاثنين د. عرفة محاضرة $\boxed{6}$

Ex: $\overline{GH}(z) = \dfrac{K(z-0.2)}{(z-1)(z+0.6)^2}$, Draw the root locus and find the range of k for stability

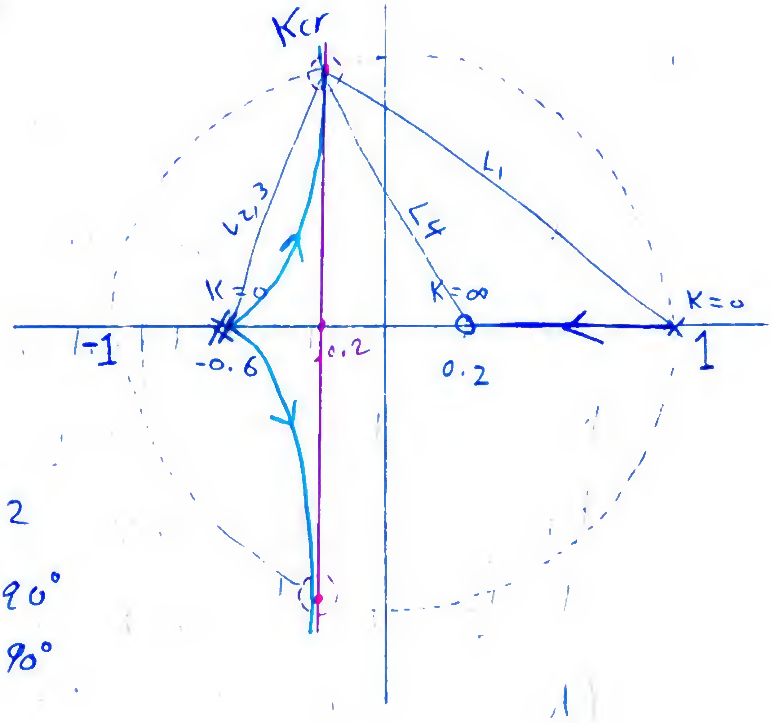
$\boxed{1}$ Poles: $n_p = 3 \Rightarrow 1, -0.6, -0.6$

Zeros: $n_z = 1 \Rightarrow 0.2$

$\boxed{2}$ z-plane

$\boxed{3}$ real part: $1 \longrightarrow 0.2$

$\boxed{4}$ Asymptotes



1- no. of asy $= n_p - n_z$

2- $\delta_c = \dfrac{\sum \text{Poles} - \sum \text{Zeros}}{n_p - n_z} = -0.2$

3- $\theta = \dfrac{(2L+1)}{n_p - n_z} \Rightarrow$ $L=0, \theta_1 \Rightarrow 90^\circ$

$L=1, \theta_2 \Rightarrow -90^\circ$

$\boxed{5}$ Determine Kcr

① $K_{cr} = \dfrac{L_1 L_{2,3}}{L_3} = \checkmark\checkmark$

② using Bilinear Transformation

Ch. eqn $\Rightarrow$ $1 + \dfrac{K(z-0.2)}{(z-1)(z+0.6)^2} = 0$

$(z-1)(z+0.6)^2 + K(z-0.2) = 0$

$(0.32 + 1.2K)r^3 + (2.56 - 1.6K)r^2 + (5.12 - 0.4K)r + 0.8K$

$\Longrightarrow$ Continue

$$\Downarrow \; > 0$$

$$\begin{array}{c|cc}
r^3 & 0.32 + 1.2K \;\; (1) & 5.12 - 0.4K \\
r^2 & 2.56 - 1.6K \;\; (2) & 0.8K \\
r^1 & A \;\; (3) & \\
r^0 & 0.8K \;\; (4) &
\end{array}$$

(1) $0.32 + 1.2K > 0$

$\Rightarrow K > -0.267$

(2) $2.56 - 1.6K > 0$

$\Rightarrow K < 1.6$

(4) $0.8K > 0 \Rightarrow K > 0$

(3) $A > 0 \Rightarrow (2.56 - 1.6K)(5.12 - 0.4K) - 0.8K(0.32 + 1.2K) > 0$

$-0.32K^2 - 9.472K + 13.167 > 0$

$K^2 + 29.6K - 40.96 < 0$

$K_{1,2} = 1.32 \; \& \; -30.92$

$(K \overset{\pm}{-} 1.32)(K \overset{\mp}{+} 30.92) < 0$

$K > 1.32 \; \& \; K < -30.92$ ✗

$K < 1.32 \; \& \; K > -30.92$ ✓

$\therefore \quad 0 < K < 1.32 \qquad K_{cr} = 1.32$

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* Bode Diagram

To study system properties in freq. domain for a discrete time system, we use bilinear Transformation, to get the system in cont. time domain.

Biliner Transformation $\boxed{z = \frac{1+r}{1-r}}$

- Remember that it's a relative stability method

given open loop Digital T.F. $\overline{GH(z)}$, to Draw the bode Diagram :

① Map from z-domain to r-domain

$$\overline{GH(z)} \longrightarrow GH(r)$$

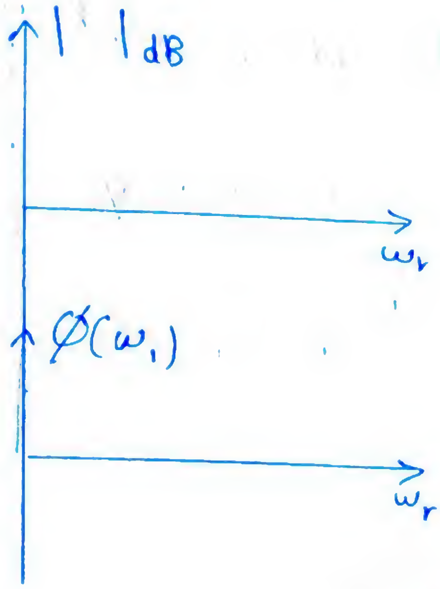
② $r \longrightarrow j\omega_r$

③ $|GH(j\omega_r)| = \dfrac{|\text{البسط}|}{|\text{مقام}|}$

$$|GH(j\omega_r)|_{dB} = 20 \log |GH(j\omega_r)|$$

④ $\phi(\omega_r) = \angle\text{البسط} - \angle\text{المقام}$; $\tan^{-1}\left(\frac{imag}{real}\right)$

⑤ Draw



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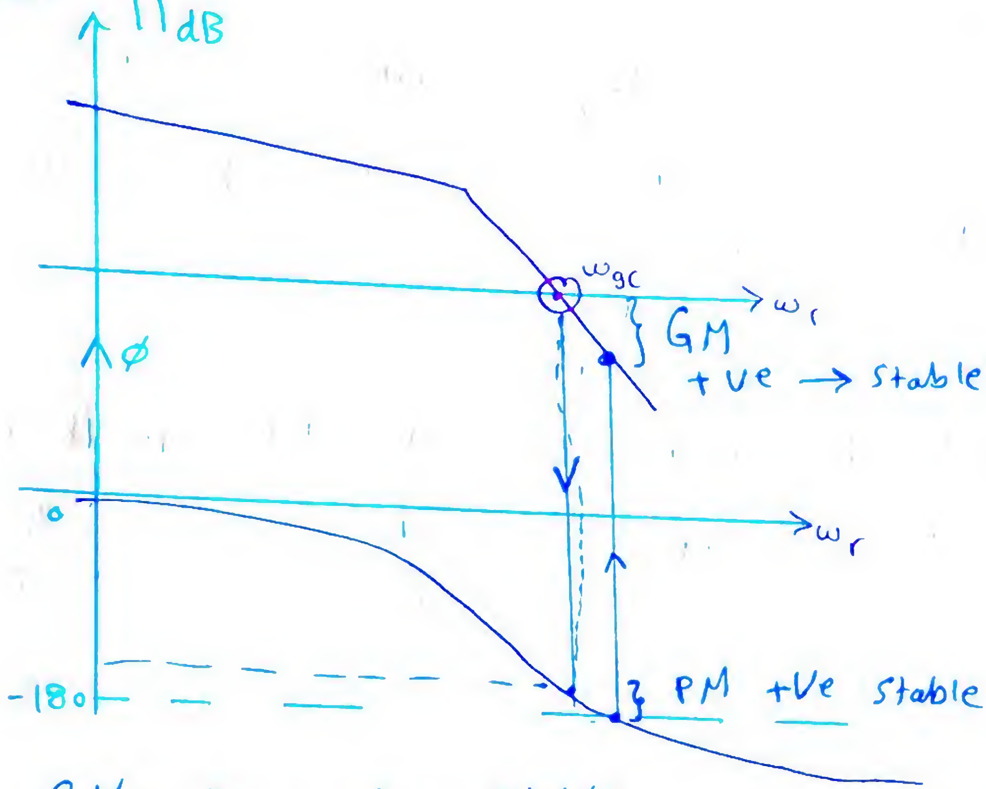
ex:



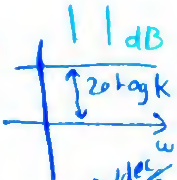
$GM_{dB} > 0 \Rightarrow$ stable ; $GM_{dB} = 0 \Rightarrow$ Critically stable

$GM_{dB} < 0 \Rightarrow$ unstable

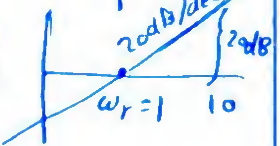
Same goes for PM

Common terms

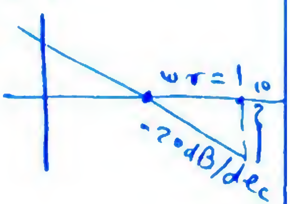
$\boxed{1}$ $GH(z) = K$; $\phi(\omega_r)$ – ;



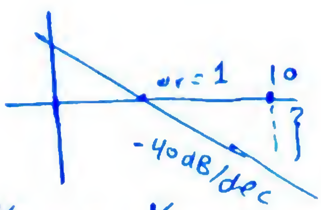
$\boxed{2}$ $s \to r \to j\omega_r$; $90°$;



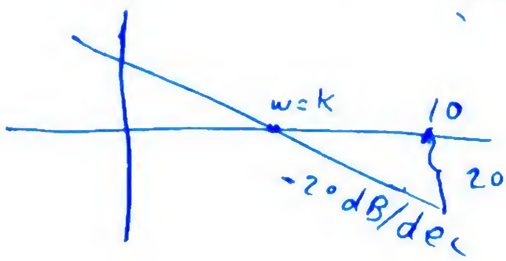
$\boxed{3}$ $\frac{1}{s} \to \frac{1}{r} \to \frac{1}{j\omega_r}$; $-90°$;



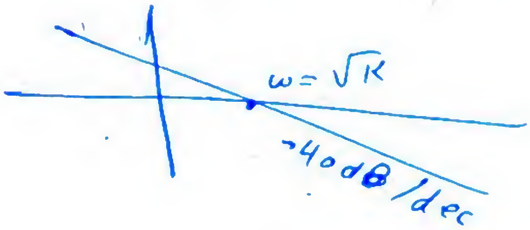
$\boxed{4}$ $\frac{1}{s^2} \to \frac{1}{r^2} \to \frac{1}{j\omega} \cdot \frac{1}{j\omega}$; $-180°$



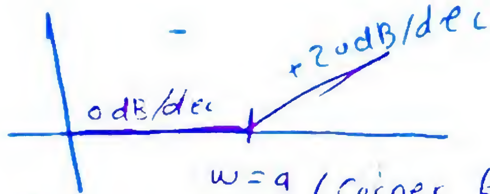
$\boxed{5}$ $GH(z) = \frac{K}{s} \to \frac{K}{r} \to \frac{K}{j\omega_r}$; $-90°$



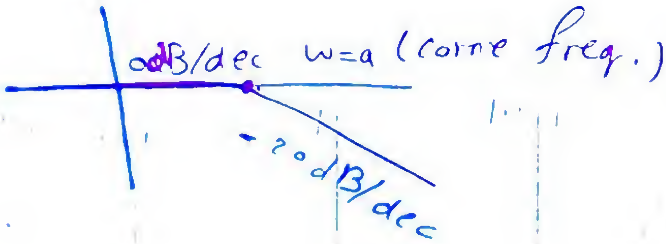
$\boxed{6}$ $\frac{K}{s^2} \rightarrow \frac{K}{r^2} \rightarrow \frac{K}{j\omega \cdot j\omega}$; $\phi(\omega_r)$ -180°



$\boxed{7}$ $\overline{GH}(z) = 1 + \frac{s}{a}$ عوض $\rightarrow 1 + \frac{r}{a} \rightarrow \left(1 + j\frac{\omega r}{a}\right)$; $\phi(\omega_r)$ $\tan^{-1}\left(\frac{\omega r}{a}\right)$



$\boxed{8}$ $GH(z) = \frac{1}{\left(1 + \frac{s}{a}\right)}$ عوض $\rightarrow \frac{1}{\left(1 + j\frac{\omega r}{a}\right)}$; $-\tan^{-1}\frac{\omega r}{a}$



Example 1:

Draw the Bode diagram for the following system

$$\overline{GH}(z) = \frac{0.5\,(z + 0.76)}{(z-1)(z - 0.45)}$$

and find ω_{gc} & ω_{pc} & GM & PM

① put $z = \frac{1+r}{1-r}$

$$GH(r) = \frac{0.5\left(\frac{1+r}{1-r} + 0.76\right)}{\left(\frac{1+r}{1-r} - 1\right)\left(\frac{1+r}{1-r} - 0.45\right)} = \frac{0.5(1-r)(1+r+0.76(1-r))}{(1+r-(1-r))(1+r-0.45(1-r))}$$

$$GH(r) = \frac{0.5(1-r)(1.76 + 0.24r)}{2r(0.55 + 1.45r)}$$

$$GH(r) = \frac{\overset{0.8}{(0.5)(1.76)}(1-r)\left(1+\frac{0.24r}{1.76}\right)}{2(0.5)r\left(1+\frac{1.45r}{0.55}\right)}$$

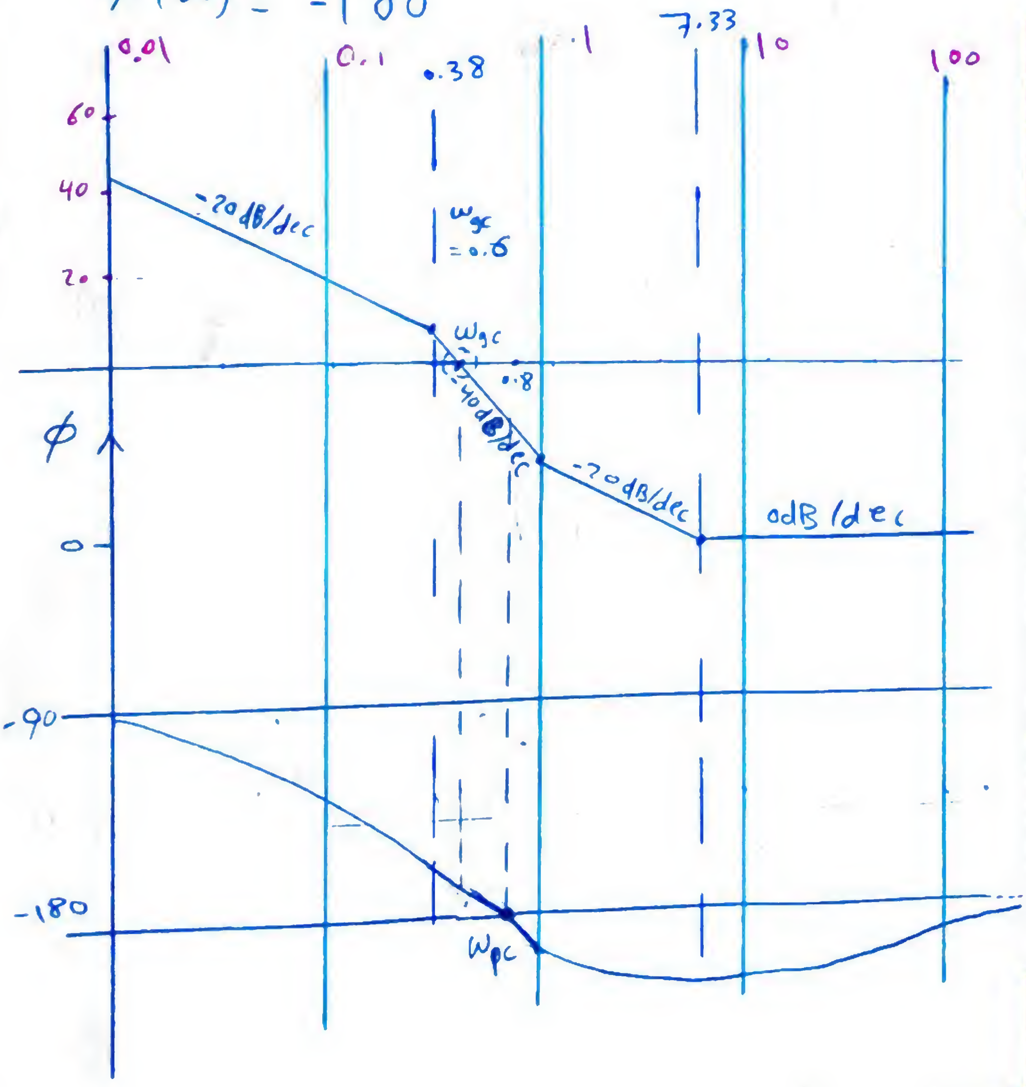
$$\boxed{= \frac{0.8(1-r)\left(1+\frac{r}{7.33}\right)}{r\left(1+\frac{r}{0.38}\right)}}$$

$$GH(j\omega_r) = \frac{0.8(1-j\omega_r)\left(1+\frac{j\omega_r}{7.33}\right)}{j\omega_r\left(1+\frac{j\omega_r}{0.38}\right)}$$

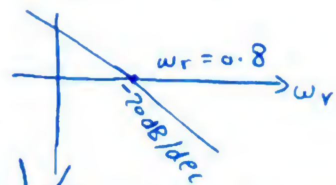
$$\phi(\omega_r) = -90 + \underbrace{\tan^{-1}(\omega_r) + \tan^{-1}\left(\frac{\omega_r}{7.33}\right)}_{\text{البسط}} - \underbrace{\tan^{-1}\left(\frac{\omega_r}{0.38}\right)}_{\text{المقام}}$$

$$\phi(0) = -90$$

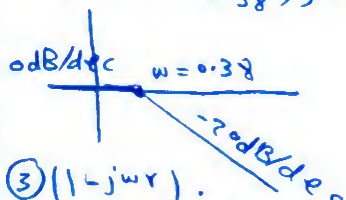
$$\phi(\infty) = -180$$



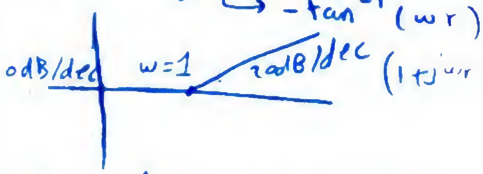
① $\frac{0.8}{j\omega_r}$; -90



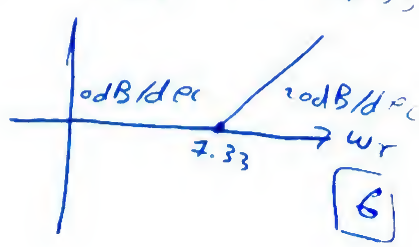
② $1/\left(1+j\frac{\omega_r}{0.38}\right)$; $-\tan^{-1}\left(\frac{\omega_r}{0.38}\right)$



③ $(1-j\omega_r)$; $\hookrightarrow -\tan^{-1}(\omega_r)$



④ $1+\frac{j\omega_r}{7.33}$; $\tan^{-1}\left(\frac{\omega_r}{7.33}\right)$



$\omega_{gc} = 0.6$

ω_r	0	0.01	0.1	0.38	0.6	1	7.33	100
ϕ	-90	-92°	-109.67	-152.8	-173.94	-196.43	-209.47	-183.4

at $\omega_r = \infty \Rightarrow \phi = -180$

عندي قيمتين للـ GM هنا قدرا أسوأ منهم وهو الأصغر